Adapting Foundation Models: From Reinforcement Learning to Multivariate Time Series Forecasting

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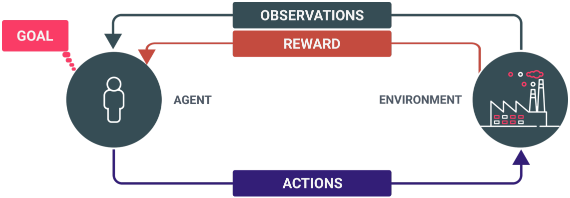
Outline

Preliminaries

Reinforcement Learning environments are Markov decision processes $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, P, r, \mu_0, \gamma \rangle$, where:

- S state space, A action space.
- Transition fn $P_t : (s, a, s') \mapsto \mathbf{Pr}(s_{t+1} = s' | s_t = s, a_t = a)$.
- Reward function $r : (s, a) \mapsto r(s, a)$.
- μ_0 initial state distribution, $\gamma \in [0, 1]$ discount factor.



Reinforcement Learning

The goal of RL is to find a policy $\pi : \mathcal{S} \to \Delta(\mathcal{A})$ that maximizes the return:

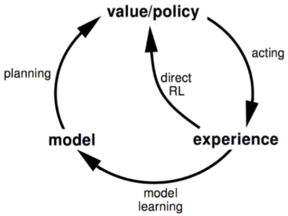
$$\eta(\pi) := \mathbb{E}_{s_0 \sim \mu_0, a_t \sim \pi,\, s_{t>0} \sim P_t}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)\right]$$

Model-based Reinforcement Learning

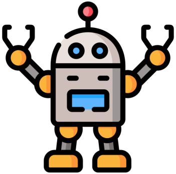
Model-based RL (MBRL) learns the transition $\hat{P}$ from interaction data. The model maximizes the log-likelihood:

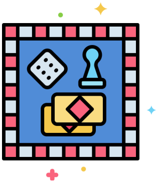
$$\mathcal{L}(\mathcal{D}; \hat{P}) = \frac{1}{N} \sum_{i=1}^{N} \log \hat{P}(s_{t+1}^i | s_t^i, a_t^i)$$

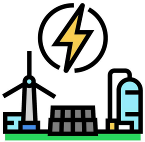
The learned model is used for policy search under the *learned* MDP $\widehat{\mathcal{M}} = \langle \mathcal{S}, \mathcal{A}, \hat{P}, r, \mu_0, \gamma \rangle$.



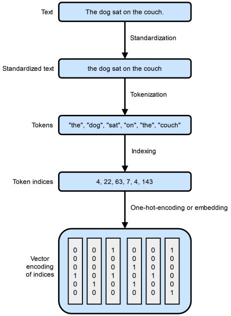
MBRL is particularly useful under **budget** and **safety** constraints.

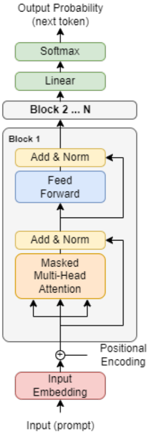


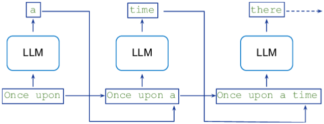




Large Language Models (LLMs) are **transformer**-based, **decoder-only** models trained using **autoregressive** next token prediction.







Numerical data tokenization

LLaMA 3 Tokenizer

- Digits: ['0', '1', ... '999']
- Token Ids: [15, 16, 17, ... 5500]

"151,167,...,267"

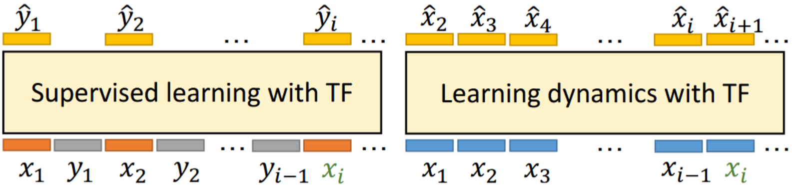
Time series processing

- Time series: [0.2513, 5.2387, 9.7889]
- Rescale+Encode: [150, 516, 850]
- Input str: '150,516,850,'
- Input str token list: ['150', ',', '516', ',', '850', ',']
- Input str token Id list: [3965, 11, 20571, 11, 16217, 11]

Sampling: *Softmax* over the digits tokens

In-context Learning (ICL)

In-context learning	Input prompt	Desired Output
Natural language processing	berry, baya, apple, manzana, banana	plátano
	Japan, mochi, France, croissant, Greece	baklava
Supervised learning $y_i = f(x_i) + \text{noise}$	$x_1, y_1, x_2, \ldots, x_{i-1}, y_{i-1}, x_i$	$f(x_i)$
Dynamical systems $x_{i+1} = f(x_i) + \text{noise}$	$x_1, x_2, x_3, \ldots, x_{i-2}, x_{i-1}, x_i$	$f(x_i)$



Problem setup

Dynamics learning using LLMs

- State space $\mathbb{R}^{d_s}$, Action space $\mathbb{R}^{d_a}$, Reward $\mathbb{R}$
- Given a trajectory

$$\tau^\pi = (s_0, a_0, s_1, a_1, s_2, \dots, s_{T-1})$$

We want to learn the distribution of the next state using ICL and a pre-trained LLM with parameters θ:

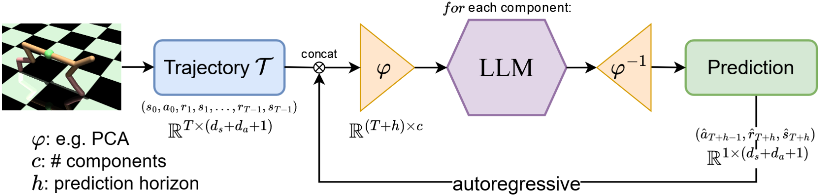
$$\left\{\hat{P}_\theta^{\pi,j}(s_t^j|\tau^\pi)\right\}_{t\leq T, j\leq d_s} = \mathsf{ICL}_\theta(\tau^\pi)$$

Challenges:

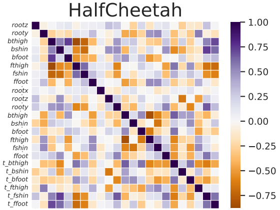
1. Multivariate states: $d_s > 1$
2. Including actions in-context: $P(s_t^j|s_0, a_0, s_1, a_1, s_2, \dots, s_{T-1})$

Approach

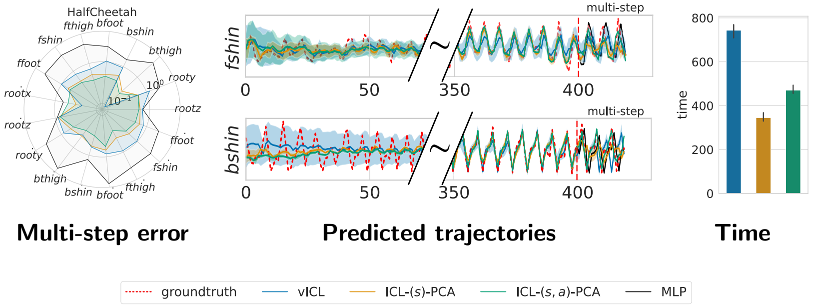
DICL: Disentangled In-Context Learning [1]



In practice, we project states and actions (s, a) into the space of `PCA` components.



Results



PCA-based DICL achieves smaller multi-step error in less computational time. We compare DICL-(s) and DICL-(s, a) using a number of components equal to half the number of features, with the vanilla approach vICL and an MLP baseline.

SAC: Soft Actor-Critic (an off-shelf RL algorithm)

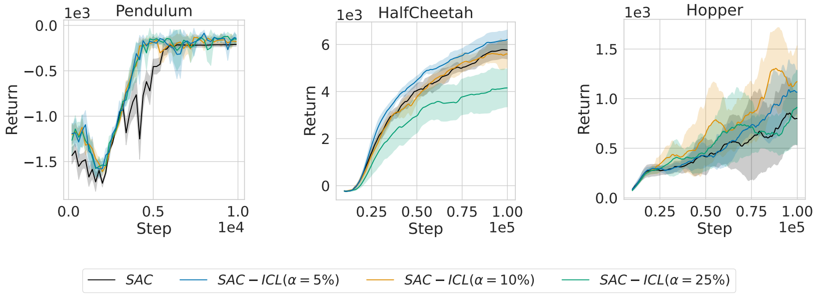
+

DICL

=

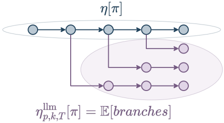
DICL-SAC

for $t = 1, \dots, N\_interactions$ **do**
New transition (s_t, a_t, r_t, s_{t+1}) from π_θ
Add (s_t, a_t, r_t, s_{t+1}) to $\mathcal{R}$
Store auxiliary action $\tilde{a}_t \sim \pi_\theta(.|s_t)$
if Generate LLM data **then**
Sample trajectory $\mathcal{T} = (s_0, \dots, s_{T_{\max}})$ from $\mathcal{R}$
$\{\hat{s}_{i+1}\}_{0 \le i \le T_{\max}} \sim$ DICL-(s) ($\mathcal{T}$)
Add $\{(s_i, \tilde{a}_i, r_i, \hat{s}_{i+1})\}_{T \le i \le T_{\max}}$ to $\mathcal{R}_{\text{llm}}$
end if
if update SAC **then**
Sample batch $\mathcal{B}$ of size b from $\mathcal{R}$
Sample batch $\mathcal{B}_{\text{llm}}$ of size $\alpha \cdot b$ from $\mathcal{R}_{\text{llm}}$
Update ϕ and ψ on $\mathcal{B} \cup \mathcal{B}_{\text{llm}}$
end if
end for



Results: DICL-SAC (Theoretical guarantee)

Under mild assumptions on the LLM prediction error ε_{llm}, we have:



Theorem (Multi-branch return bound)

- T *the context length*
- $p \in [0, 1]$ *probability of branching*
- k *the branch length*
- ε_{llm} *the LLM in-context learning prediction error*

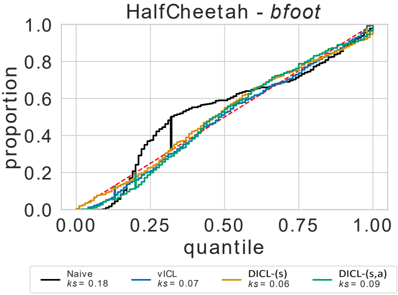
$$|\eta(\pi) - \eta^{llm}_{p,k,T}(\pi)| \leq 2\frac{\gamma^T}{1-\gamma} r_{\max} k^2\, p\, \varepsilon_{llm}(T)$$

where $r_{\max} = \max_{s\in\mathcal{S},a\in\mathcal{A}} r(s, a)$.

Results: Calibration

Quantile calibration: For probabilistic regression, a perfectly calibrated forecaster means that $p\%$ of groundtruth values fall within the $p\%$-confidence interval of the predicted CDF.

LLMs are well-calibrated in-context forecasters.



Outline

Problem setup

Problem setup

Consider a multivariate time series forecasting task:

- $\mathbf{X} \in \mathbb{R}^{L \times D}$ data matrix
- $\mathbf{Y} \in \mathbb{R}^{H \times D}$ target
  - L lookback window (context length)
  - H forecasting horizon
  - D dimension (number of covariates)

We want to find the best adapter φ^* such that:

Definition (adapter)

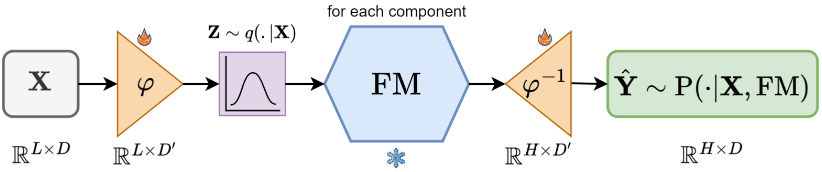
Feature-space transformation $\varphi : \mathbb{R}^D \to \mathbb{R}^{D'}$ such that:

$$\hat{\mathbf{Y}}(\mathbf{X}; \varphi) = \varphi^{-1}\big(\texttt{FM}(\varphi(\mathbf{X}))\big), \text{ and } \varphi^* = \text{argmin}_\varphi \|\mathbf{Y} - \hat{\mathbf{Y}}(\mathbf{X}; \varphi)\|_{\mathrm{F}}^2,$$

where FM is a fixed time series foundation model.

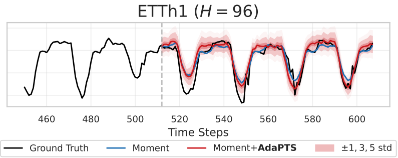
Approach

AdaPTS: Adapters for Probabilistic multivariate Time Series forcasting [2]



Properties:

1. Mixing features
2. Probabilistic predictions



Results

Results: Forecasting error (MSE)

Families of adapters:

1. deterministic
   - Linear AutoEncoder
   - Deep non-linear AutoEncoder
   - Normalizing Flow
2. probabilistic
   - + Variational Inference
   - + MC Dropout

Dataset	H	No adpt	with adapter				
		Moment	PCA	LinAE	dropLAE	LinVAE	VAE
ETTh1	96	$0.411_{\pm.012}$	$0.433_{\pm.001}$	$0.402_{\pm.002}$	$\mathbf{0.395_{\pm.003}}$	$0.400_{\pm.001}$	$0.404_{\pm.001}$
	192	$\mathbf{0.431_{\pm.001}}$	$0.440_{\pm.000}$	$0.452_{\pm.002}$	$0.446_{\pm.001}$	$0.448_{\pm.002}$	$\mathbf{0.431_{\pm.001}}$
Ill	24	$2.902_{\pm.023}$	$2.98_{\pm.001}$	$2.624_{\pm.035}$	$2.76_{\pm.061}$	$2.542_{\pm.036}$	$\mathbf{2.461_{\pm.008}}$
	60	$3.000_{\pm.004}$	$3.079_{\pm.000}$	$3.110_{\pm.127}$	$2.794_{\pm.015}$	$\mathbf{2.752_{\pm.040}}$	$2.960_{\pm.092}$
Wth	96	$0.177_{\pm.010}$	$0.176_{\pm.000}$	$0.169_{\pm.000}$	$\mathbf{0.156_{\pm.001}}$	$0.161_{\pm.001}$	$0.187_{\pm.001}$
	192	$0.202_{\pm.000}$	$0.208_{\pm.001}$	$\mathbf{0.198_{\pm.001}}$	$0.200_{\pm.001}$	$0.204_{\pm.000}$	$0.226_{\pm.000}$
ExR	96	$\mathbf{0.130_{\pm.011}}$	$0.147_{\pm.000}$	$0.167_{\pm.013}$	$\mathbf{0.130_{\pm.011}}$	$0.243_{\pm.039}$	$0.455_{\pm.010}$
	192	$\mathbf{0.210_{\pm.002}}$	$0.222_{\pm.000}$	$0.304_{\pm.005}$	$0.305_{\pm.013}$	$0.457_{\pm.020}$	$0.607_{\pm.021}$

Results: Interpretable latent representations

Desirable representation learning properties:

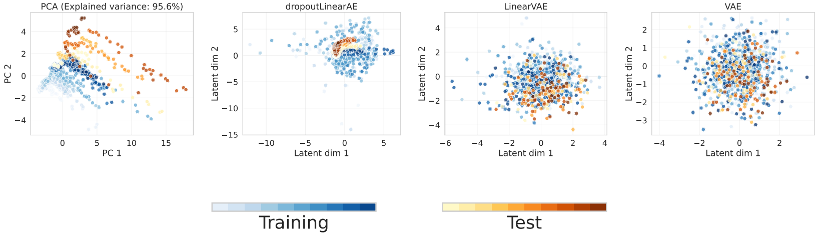
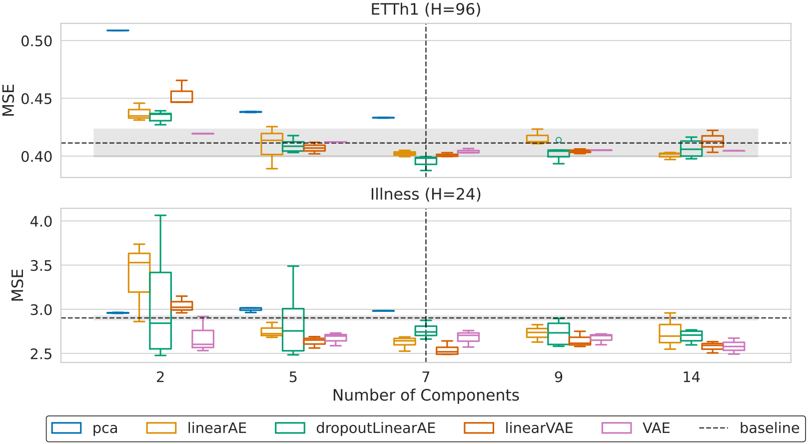


Figure: Visualization of the latent representation obtained by different adapters on Illness($H = 24$). Shaded colors indicate the time dimension, with lighter colors representing earlier timesteps.

Results: Dimensionality reduction

Better forecasting accuracy even with lower dimensions



Outline

Conclusion

- We presented **DICL**, a methodology to adapt LLMs for the task of dynamics learning in MBRL
- We then presented **AdaPTS** a learning-based and probabilistic extension of adapters to multivariate time series forecasting

Take Home Message

Foundation Models are powerful predictors trained on vast amounts of data
$\rightarrow$ **Adapters** are an effectve way to adapt FMs to custom problems

References

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Thank You!

Want to know more?

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Slides available at:

https://abenechehab.github.io/assets/pdf/adapters.pdf